MATH 2B/5B Prep: Trigonometric Functions

1. Show the trig identity $1+\cot^2(x)=\csc^2(x)$. (See Example 1)

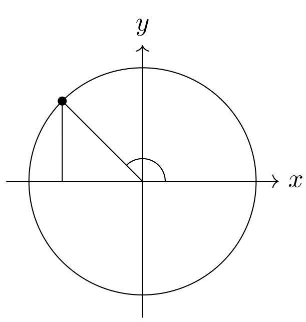
Solution: Start with the trig identity $\sin^2(x)+\cos^2(x)=1$, then divide by $\sin^2(x)$. This gets

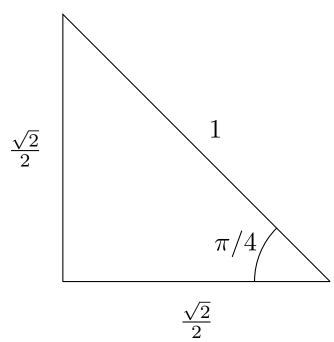
$$\frac{\sin^2(x)}{\sin^2(x)}+\frac{\cos^2(x)}{\sin^2(x)}=\frac{1}{\sin^2(x)}$$

which simplifies to $1+\cot^2(x)=\csc^2(x)$.

2. Compute $\cos(3\pi/4)$ and $\sin(3\pi/4)$.

Solution:



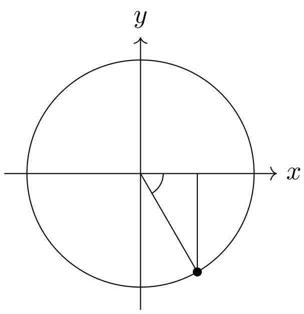


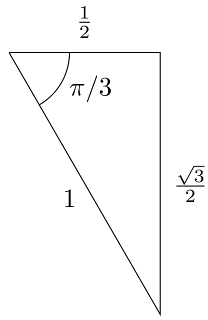
The picture on the left illustrates the location of this point on the unit circle corresponding to the angle $3\pi/4$. Notice the triangle that forms has an angle of $\pi/4$ from the negative x-axis. The right image shows that this triangle has sides of length $\sqrt{2}/2$, which is both $\cos(\pi/4)$ and $\sin(\pi/4)$. Since the point lands in the second quadrant we know the cosine value is negative and the sine value is positive. Therefore

$$\cos(3\pi/4)=-\frac{\sqrt{2}}{2}\qquad\qquad \sin(3\pi/4)=\frac{\sqrt{2}}{2}$$

3. Find $\cos(-\pi/3)$ and $\sin(-\pi/3)$.

Solution:





Since we have a negative angle we go clockwise (not counter-clockwise) around the unit $\pi/3$ radians and thus end in the fourth quadrant. Again in the left diagram we see the related point on the unit circle and the triangle that is formed by adding a line to the positive x-axis. On the right is a larger image of this triangle - we get an angle of $\pi/3$ which gives one side of length $1/2$ and one of $\sqrt{3}/2$, corresponding to $\cos(\pi/3)$ and $\sin(\pi/3)$ respectively. Since the point is in the fourth quadrant this means the cosine value is positive and the sine value is negative. Hence

$$\cos(-\pi/3) = \frac{1}{2} \qquad \sin(-\pi/3) = -\frac{\sqrt{3}}{2}$$

4. Write $\cos^2(x)$ in terms of $\cos(2x)$.

Solution: Recall the double angle formula $\cos(2x) = 2\cos^2(x) - 1$. Adding 1 to both sides gives $2\cos^2(x) = \cos(2x) + 1$. Dividing by 2 gives

$$\cos^2(x) = \frac{1}{2}(\cos(2x) + 1)$$